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Model of factory with alternative sources used to reduce power supply

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***Abstract* — In this paper mathematical model for equivalent circuit of factory using alternative sources to reduce power supply is created. Equivalent circuit is designed for steady-state analysis. Model is built using the Sparse Tableau Analysis and is designed for logic programmable controllers programs to provide mentioned factory operation.**

***Keywords* — Alternative sources, steady state model, Sparse Tableau Analysis, reduce power supply**

I. Introduction

Equivalent circuit designed for factories with alternative sources that are used solely for factory power supply, for steady state analyzes is shown in Fig. 1. Equivalent circuit has been designed based on the problems formulated for this type of industrial plant operation and the proposed operating conditions to address these issues outlined in [1]. Based on this design a model with linear passive elements and non-harmonic sources for steady state analysis was created.

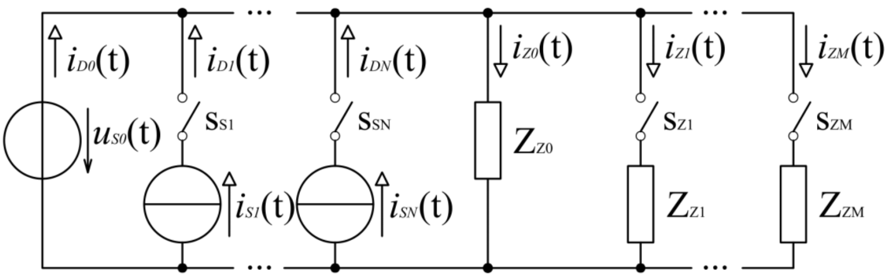

Fig. 1 Equivalent circuit of industrial factory with alternative sources for steady state analysis.

The power ratios of equivalent circuit can be analyzed by comparing the DC power of the 0th harmonics and comparing the apparent power or active and reactive power of the other harmonics. Another option is to determine power ratios by comparing all currents harmonics of each element of equivalent circuit. Electrical circuit simulation methods such as Modified Nodal Analysis (MNA) [2] or Sparse Tableau Analysis (STA) [3] utilize element voltages and currents and node voltages. For this reason we will use the voltages and currents of harmonics to analyze the power ratios.

II. Simulation model of factory created by Sparse tableau Analysis

We use STA to create a simulation model consisting of a system of equations describing equivalent circuit. In the described circuit there are linear passive elements or elements with electrical values linearly or nonlinearly dependent on non-electrical parameters (powers or alternate current currents). In the case of nonlinear dependence we can use the approximation [4]. This is possible if the simplification of the function does not have a significant impact on the result of calculation [5]. Then a linear equations system created by STA (hereinafter STA system) can be used to describe [6].

This system can be expressed in two shapes. The STA system (1) could be called a complete one, and the STA system (2) could be called a simplified one. Matrix **A** is a (reduced) node incidence matrix, **E** is matrix with 1 in main diagonal, **Z** is an impedance matrix, and **Y** is an admittance matrix. Other elements are column vectors and **i** is a vector of branch currents, **u** is a vector of branch stresses, **v** is a vector of node voltages and **s** is a vector of voltage and currents sources. Simplified STA system will be used for the description because it contains a smaller number of equations.

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} & -\mathbf{A}^T \\ \mathbf{Z} & \mathbf{Y} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{s} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{Z} & \mathbf{Y}\mathbf{A}^T \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{s} \end{bmatrix} \quad (2)$$

Described circuit contains non-harmonic sources (we consider the final number of harmonics of all non-harmonic sources) we will build the STA system separately for each harmonics. Consequently, we will combine crated STA systems into one equations system that will describe all used harmonics. We will now explain the general procedure for constructing an STA system of k-th harmonics of equivalent circuit where k is from range <0;g>. Electrical circuit describing every harmonics, auxiliary elements and markings for assembling the STA system are shown in Fig. 2.

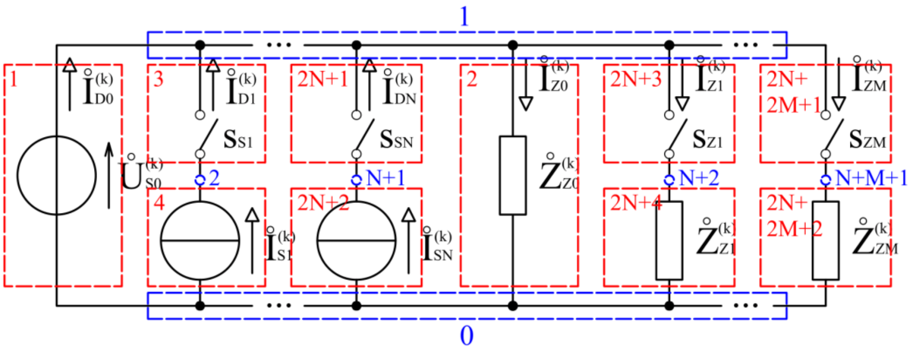

Fig. 2 Any harmonic component in equivalent circuit designed to create a model using STA.

The number of nodes in circuit is $M+N+2$ of which independent nodes is $M+N+1$ and the number of branches (corresponding to the number of elements) is $2(M+N+1)$. Each separate circuit element is a branch in the simulation [6]. STA system for each harmonics (3) will consist of $3(M+N+1)$ linear equations with $3(M+N+1)$ unknown values, with $2(M+N+1)$ unknown branch currents and $(M+N+1)$ unknown node voltages.

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{Z}^{(k)} & \mathbf{Y}^{(k)}\mathbf{A}^T \end{bmatrix} \begin{bmatrix} \mathbf{i}^{(k)} \\ \mathbf{v}^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{s}^{(k)} \end{bmatrix} \quad (3)$$

Node incidence matrix **A** corresponds to the connection of circuit elements. Because elements connection of each harmonics does not change, the matrix **A** (4) will be the same for each used harmonics.

$$\mathbf{A} = \begin{array}{c|cccccccccccc} & 1 & 2 & 3 & 4 & \cdots & 2N+1 & 2N+2 & 2N+3 & 2N+4 & \cdots & 2N+2M+1 & 2N+2M+2 \\ \hline 1 & -1 & 0 & -1 & 0 & \cdots & -1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 2 & 0 & 1 & 1 & -1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ N+1 & 0 & 0 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ N+2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ M+N+1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{array} \quad (4)$$

Elements outside the main diagonal in square matrixes $\mathbf{Z}^{(k)}$ and $\mathbf{Y}^{(k)}$ are zero. Therefore, Table I lists only the main diagonal elements of the impedance and admittance matrix. It is possible to assemble these matrices for each considered harmonics according to Table I.

Table I
Elements of main diagonal of impedance and admittance matrix

Main diagonal elements of impedance matrix $\mathbf{Z^{(K)}}$	Main diagonal elements of admittance matrix $\mathbf{Y^{(K)}}$
$Z_{1,1}^{(k)}=0$	$Y_{1,1}^{(k)}=1$
$Z_{2,2}^{(k)}=-\mathbf{Z_{Z0}^{(k)}}$	$Y_{2,2}^{(k)}=1$
$Z_{3,3}^{(k)}=\langle\begin{matrix}1 & s_{S1}\ \text{on}\\ 0 & s_{S1}\ \text{off}\end{matrix}$	$Y_{3,3}^{(k)}=\langle\begin{matrix}0 & s_{S1}\ \text{on}\\ 1 & s_{S1}\ \text{off}\end{matrix}$
$Z_{4,4}^{(k)}=1$	$Y_{4,4}^{(k)}=0$
...	...
$Z_{2N+1,2N+1}^{(k)}=\langle\begin{matrix}1 & s_{SN}\ \text{on}\\ 0 & s_{SN}\ \text{off}\end{matrix}$	$Y_{2N+1,2N+1}^{(k)}=\langle\begin{matrix}0 & s_{SN}\ \text{on}\\ 1 & s_{SN}\ \text{off}\end{matrix}$
$Z_{2N+2,2N+2}^{(k)}=1$	$Y_{2N+2,2N+2}^{(k)}=0$
$Z_{2N+3,2N+3}^{(k)}=\langle\begin{matrix}1 & s_{Z1}\ \text{on}\\ 0 & s_{Z1}\ \text{off}\end{matrix}$	$Y_{2N+3,2N+3}^{(k)}=\langle\begin{matrix}0 & s_{Z1}\ \text{on}\\ 1 & s_{Z1}\ \text{off}\end{matrix}$
$Z_{2N+4,2N+4}^{(k)}=-\mathbf{Z_{Z1}^{(k)}}$	$Y_{2N+4,2N+4}^{(k)}=1$
...	...
$Z_{2N+2M+1,2N+2M+1}^{(k)}=\langle\begin{matrix}1 & s_{ZM}\ \text{on}\\ 0 & s_{ZM}\ \text{off}\end{matrix}$	$Y_{2N+2M+1,2N+2M+1}^{(k)}=\langle\begin{matrix}0 & s_{ZM}\ \text{on}\\ 1 & s_{ZM}\ \text{off}\end{matrix}$
$Z_{2N+2M+2,2N+2M+2}^{(k)}=-\mathbf{Z_{ZM}^{(k)}}$	$Y_{2N+2M+2,2N+2M+2}^{(k)}=1$

Vectors $\mathbf{i^{(k)}}$ and $\mathbf{v^{(k)}}$ are unknown for the equation system. Elements of vector $\mathbf{i^{(k)}}$ represent currents flowing through all elements of the equivalent circuit. Vector elements $\mathbf{v^{(k)}}$ correspond to voltages of independent nodes. The vector $\mathbf{s^{(k)}}$ represents the values of known voltage and current sources. Way of creating vector $\mathbf{s^{(k)}}$ and the vector elements in $\mathbf{i^{(k)}}$ and $\mathbf{v^{(k)}}$ corresponding to the currents and voltages of elements for any harmonics are shown in (5).

$$\mathbf{i^{(k)}}=\begin{bmatrix} i_1^{(k)}\\ i_2^{(k)}\\ i_3^{(k)}\\ i_4^{(k)}\\ \vdots\\ i_{2N+1}^{(k)}\\ i_{2N+2}^{(k)}\\ i_{2N+3}^{(k)}\\ i_{2N+4}^{(k)}\\ \vdots\\ i_{2N+2M+1}^{(k)}\\ i_{2N+2M+2}^{(k)}\end{bmatrix}=\begin{bmatrix}\mathbf{I_{D0}^{(k)}}\\ \mathbf{I_{Z0}^{(k)}}\\ \mathbf{I_{D1}^{(k)}}\\ \mathbf{I_{D1}^{(k)}}\\ \vdots\\ \mathbf{I_{DN}^{(k)}}\\ \mathbf{I_{DN}^{(k)}}\\ \mathbf{I_{Z1}^{(k)}}\\ \mathbf{I_{Z1}^{(k)}}\\ \vdots\\ \mathbf{I_{ZM}^{(k)}}\\ \mathbf{I_{ZM}^{(k)}}\end{bmatrix}\quad \mathbf{v^{(k)}}=\begin{bmatrix} v_1^{(k)}\\ v_2^{(k)}\\ \vdots\\ v_{N+1}^{(k)}\\ v_{N+2}^{(k)}\\ \vdots\\ v_{M+N+1}^{(k)}\end{bmatrix}=\begin{bmatrix}\mathbf{U_1^{(k)}}\\ \mathbf{U_2^{(k)}}\\ \vdots\\ \mathbf{U_{N+1}^{(k)}}\\ \mathbf{U_{N+2}^{(k)}}\\ \vdots\\ \mathbf{U_{M+N+1}^{(k)}}\end{bmatrix}\quad \mathbf{s^{(k)}}=\begin{bmatrix} s_1^{(k)}\\ s_2^{(k)}\\ s_3^{(k)}\\ s_4^{(k)}\\ \vdots\\ s_{2N+1}^{(k)}\\ s_{2N+2}^{(k)}\\ s_{2N+3}^{(k)}\\ s_{2N+4}^{(k)}\\ \vdots\\ s_{2N+2M+1}^{(k)}\\ s_{2N+2M+2}^{(k)}\end{bmatrix}=\begin{bmatrix}\mathbf{U_{S0}^{(k)}}\\ 0\\ 0\\ \mathbf{I_{S1}^{(k)}}\\ \vdots\\ 0\\ \mathbf{I_{SN}^{(k)}}\\ 0\\ 0\\ \vdots\\ 0\\ 0\end{bmatrix} \tag{5}$$

It is necessary to include parameters that affect alternative resources current (power) in the equation system. We will consider that for every current harmonics of the current source (represents a separately switched part of alternative source) can be applied (6) where k is from range $<0;g>$, e is from range $<1;N>$ and h represents the number of parameters affecting the given component.

$$\mathbf{I_{Se}^{(k)}}=f_{Se}^{(k)}\left(x_1,\ \dots\ ,x_h\right) \tag{6}$$

The vector $\mathbf{s^{(k)}}$ represents the known values of voltage and current sources. After adding of parameters affecting the individual harmonics of parts alternative sources currents, for any harmonics we can create vector $\mathbf{s^{(k)}}$ according to (7). It is necessary to realize that x_1 to x_h are known, because the vector s (k) must contain only known values.

$$\mathbf{s^{(k)}} = \begin{bmatrix} s_1^{(k)} \\ s_2^{(k)} \\ s_3^{(k)} \\ s_4^{(k)} \\ \vdots \\ s_{2N+1}^{(k)} \\ s_{2N+2}^{(k)} \\ s_{2N+3}^{(k)} \\ s_{2N+4}^{(k)} \\ \vdots \\ s_{2N+2M+1}^{(k)} \\ s_{2N+2M+2}^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{U_{S0}^{(k)}} \\ 0 \\ 0 \\ f_{S1}^{(k)}\left(x_1 , \dots , x_h\right) \\ \vdots \\ 0 \\ f_{SN}^{(k)}\left(x_1 , \dots , x_h\right) \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

After crating of all matrixes and vectors containing known values (i.e. except $\mathbf{i^{(0)}}$ to $\mathbf{i^{(g)}}$ and $\mathbf{v^{(0)}}$ to $\mathbf{v^{(g)}}$)) for each harmonics, we get a complete linear STA equation system for equivalent circuit for the final harmonics (8). This equations system will consist of *3.(g+1).(M+N+1)* linear equations with *3.(g+1).(M+N+1)* unknown values.

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{Z^{(0)}} & \mathbf{Y^{(0)}A^T} \\ \mathbf{A} & \mathbf{0} \\ \mathbf{Z^{(1)}} & \mathbf{Y^{(1)}A^T} \\ & \vdots \\ \mathbf{A} & \mathbf{0} \\ \mathbf{Z^{(g)}} & \mathbf{Y^{(g)}A^T} \end{bmatrix} \begin{bmatrix} \mathbf{i^{(0)}} & \mathbf{i^{(1)}} & \cdots & \mathbf{i^{(g)}} \\ \mathbf{v^{(0)}} & \mathbf{v^{(1)}} & & \mathbf{v^{(g)}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{s^{(0)}} \\ \mathbf{0} \\ \mathbf{s^{(1)}} \\ \vdots \\ \mathbf{0} \\ \mathbf{s^{(g)}} \end{bmatrix} \tag{8}$$

Now assembled set of equations can be solved. After solving it values of the vector elements $\mathbf{i^{(0)}}$ to $\mathbf{i^{(g)}}$ and the vectors $\mathbf{v^{(0)}}$ to $\mathbf{v^{(g)}}$ are obtained. If it is necessary to determine the performance of the individual elements, proceed as follows. We determine vectors of each element voltage for each harmonics $\mathbf{u^{(0)}}$ to $\mathbf{u^{(g)}}$, according to (9), where k is from range <0;*g*>.

$$\left[\mathbf{u^{(k)}}\right] = \left[\mathbf{A^T}\right].\left[\mathbf{v^{(k)}}\right] \tag{9}$$

Subsequently, we determine the active and reactive power of elements using the vectors $\mathbf{i^{(0)}}$ to $\mathbf{i^{(g)}}$ and $\mathbf{u^{(0)}}$ to $\mathbf{u^{(g)}}$. The active power of the element is determined by (10) and similarly the reactive power of element is determined by (11). In (10) and (11), h is line in the respective vectors and is from range <1;*2M+2N+2*>, u_h and i_h are values of vectors for each harmonic component in line h.

$$P_h = u_h^{(0)}.i_h^{(0)} + \sum_{k=1}^{g} \left|u_h^{(k)}\right|.\left|i_h^{(k)}\right|.\cos\left(\varphi_{u_h^{(k)}} - \varphi_{i_h^{(k)}}\right) \tag{10}$$

$$Q_h = \sum_{k=1}^{g} \left|u_h^{(k)}\right|.\left|i_h^{(k)}\right|.\sin\left(\varphi_{u_h^{(k)}} - \varphi_{i_h^{(k)}}\right) \tag{11}$$

The value of apparent power of the element is determined by the effective value of the non-harmonic voltage and current of the element. The apparent power of any element is determined by (12), where h is the line in the respective vectors, u_h and i_h are the values of the vectors in line h.

$$S_h = \sqrt{u_h^{(0)2} + u_h^{(1)2} + \ldots + u_h^{(g)2}}.\sqrt{i_h^{(0)2} + i_h^{(1)2} + \ldots + i_h^{(g)2}} \tag{12}$$

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