JIEE Časopis priemyselnej elektrotechniky
Journal of Industrial Electrical Engineering
ISSN 2454-0900
www.jiee.eu

Different geometries of spiral planar inductors

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***Abstract* — This paper deals with different geometries of spiral planar inductors. The spiral planar inductors are presented in different geometrical shapes: circular, square, octagonal, and hexagonal. The article describes three methods for obtaining the value of inductance. The first method is called a modified Wheeler's formula, the second is called Monomial formula or data fitted Monomial expression and the last Mohan's formula or also called expression based on current sheet approximation.**

***Keywords* —approximation, geometries, inductance, spiral planar inductor**

I. INTRODUCTION

Inductance is a measure of the distribution of the magnetic field near and inside a current carrying conductor. It is a property of the physical layout of the conductor and is a measure of the ability of the conductor to link magnetic flux, or store magnetic energy. Magnetic energy storage circuit elements are known as inductors. Such inductive elements come in a variety of shapes and sizes, ranging from toroids and solenoids for relatively large scale circuits, to monolithic structures for integrated circuits. An example of the monolithic type is a planar micro strip spiral inductor which is an integral part of many radio frequency (RF) and microwave frequency circuits [1]. The effects that limit a spiral inductor's performance at high frequencies are as follows:

1. Electric field penetration into the substrate.
2. Skin effect—current redistribution within the metal conductor cross section.
3. Proximity effect—current redistribution due to neighboring current carrying conductors.
4. Magnetic field penetration into the substrate.

The first effect is caused by time-varying electric fields whereas the remaining three are due to their time-varying magnetic fields. Since spiral inductors are the vital part of many RF circuits, an accurate model for micro strip spiral inductors can accurately predict the device performance. Greenhouse, Wheeler and Mohan have developed simple algorithms to estimate the inductance of planar rectangular spirals. The parasitic reactance, conductor and substrate losses and its frequency dependence are also included in [1].

Planar spiral inductors have limited Q's, but have inductances that are well-defined over a broad range of frequency variations. Square or rectangular spirals are popular because of the ease of their layout and analysis. However, other polygonal spirals are also used in RF circuits. Square or rectangular spirals have lower self resonant frequency (SRF). Polygons with more than four sides improve performance. Among these, hexagonal and octagonal inductors are widely used [1].

The spiral planar inductors are presented in different geometrical shapes, circular, square, octagonal, and hexagonal. They all are characterized by the same geometrical parameters: the number of turns, the internal diameter, the external diameter of the inductor, the width and the thickness of the rectangular coil, and the spacing of inter-whorls. The thickness of conductor material has only a very small effect on inductance value [2].

II. GEOMETRIES OF SPIRAL PLANAR COIL

Square spirals are popular because of the ease of their layout. However, other polygonal spirals have also been used in circuit design. Some designers prefer polygons with more than four sides to improve performance. Among these, hexagonal and octagonal inductors are used widely [4]. Fig. 1(a)–(d) shows the layout for square, hexagonal, octagonal, and circular inductors, respectively.

We have several types of plane inductor geometry that we can integrate. In the literature, we can find several analytical expressions such as Bryan's formula, Wheeler's formula, Mohan's formula etc. These expressions enable us to make a comparison of the value of inductance for various spiral geometries: square, circular, hexagonal or octagonal.

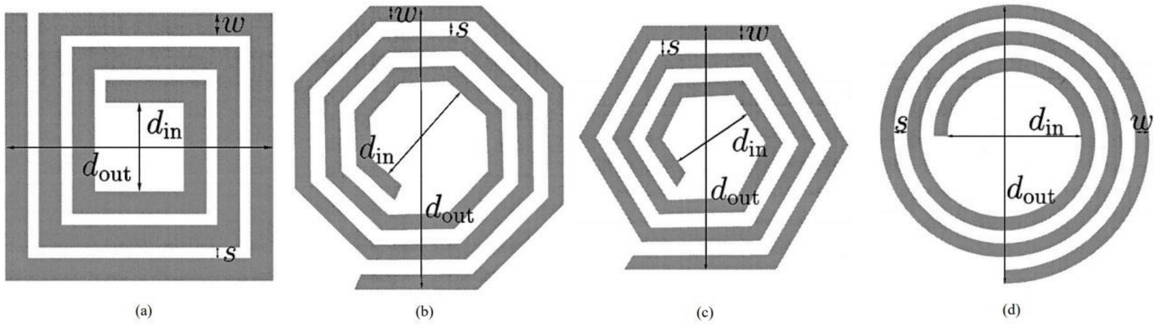

Fig. 1 Type of inductor realization: (a) square, (b) hexagonal, (c) octagonal, (d) circular

A. Modified Wheeler's formula

Wheeler's expression allows an evaluation of the inductance of an hexagonal, octagonal or square reel carried out in a discrete way. A simplification can be operated in the integrated planar case [3] Inductance given by the Wheeler's method has as an equation (1):

$$L_{mw} = k_1 \mu_0 \mu_r \frac{n^2 d_{avg}}{1 + k_2 A_m}, \tag{1}$$

where A_m is the fill ratio, defined by the equation (2):

$$A_m = \frac{d_{out} - d_{in}}{d_{out} + d_{in}}. \tag{2}$$

The ratio represents how hollow the inductor is: for small we have a hollow inductor ($d_{out} \approx d_{in}$) and for a large we have a full inductor ($d_{out} \gg d_{in}$). Two inductors with the same average diameter but different fill ratios will, of course, have different inductance values. The full one has a smaller inductance because its inner turns are closer to the center of the spiral and so contribute less positive mutual inductance and more negative mutual inductance [4].

Next d_{avg} is the average diameter of coil winding defined by the internal diameter din, and the external diameter d_{out} by equation (3):

$$d_{avg} = \frac{d_{out} + d_{in}}{2}, \tag{3}$$

k_1 and k_2 are two coefficients which depend on the used geometrical form. The values of these two coefficients are given in Table 1.

Table 1
Values of the coefficients used in Wheeler's formula

Form	k_1	k_1
Square	2.34	2.75
Hexagonal	2.33	3.82
Octagonal	2.25	3.55

We can consider in Wheeler's formula, that the value of inductance is proportional to ratio k_1/k_2 shown in the Table 2.

Table 2
Values of ratio k_1/k_2 according to the geometry.

Geometry	Square	Hexagonal	Octagonal
Rapport k_1/k_2	0.85	0.82	0.63

The greatest value of the ratio k_1/k_2 is allotted to the square geometry.

B. Monomial formula

The Monomial expression [4] used to calculate the inductance is based on the equation (4):

$$L_{mon} = \beta d_{out}^{\alpha_1} w^{\alpha_2} d_{avg}^{\alpha_3} n^{\alpha_4} s^{\alpha_5}, \tag{4}$$

where $\beta, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are coefficients which depend on the geometrical form of the spiral planar inductor. We can see the values of these coefficients in Table 3.

Table 3
Values of the coefficients used in the monomial formula

Geometry	Square	Hexagonal	Octagonal
$\boldsymbol{\beta}$	1.62×10^{-3}	1.28×10^{-3}	1.33×10^{-3}
$\boldsymbol{\alpha_1\ (d_{out})}$	-1.21	-1.24	-1.21
$\boldsymbol{\alpha_2\ (w)}$	-0.147	-0.147	-0.163
$\boldsymbol{\alpha_3 (d_{avg})}$	2.4	2.47	2.43
$\boldsymbol{\alpha_4}$ **(n)**	1.78	1.77	1.75
$\boldsymbol{\alpha_5 (s)}$	-0.03	-0.049	-0.049

for α_1, α_2 and α_5 having negative values, we can consider that the value of the inductance is proportional to the ratio $(\beta\alpha_3\alpha_4)/(\alpha_1\alpha_2\alpha_5)$ in Table 4.

Table 4
Value of ratio $(\beta\alpha_3\alpha_4)/(\alpha_1\alpha_2\alpha_5)$ according to the geometry

Geometry	Ratio
Square	1296.9472
Hexagonal	529.313
Octagonal	586.23

We also notice that the greatest value of ratio $(\beta\alpha_3\alpha_4)/(\alpha_1\alpha_2\alpha_5)$ is allotted to the square geometry.

C. Mohan's formula

Another simple and accurate expression for the inductance of a spiral planar inductor can be obtained by approximating the sides of the spirals by symmetrical current sheets of equivalent current densities. For example, in the case of the square, we obtain four identical current sheets. The current sheets on opposite sides are parallel to one another, whereas the adjacent ones are orthogonal. Using symmetry and the fact that sheets with orthogonal current sheets have zero mutual inductance, the computation of the inductance is now reduced to evaluating the self-inductance of one sheet and the mutual inductance between opposite current sheets [4]. The resulting expression given by Mohan for the calculation of inductance is (5) [5]:

$$L_{mw} = \frac{\mu N^2 d_{avg} C_1}{2}\left[\left(\ln\frac{C_2}{g}\right) + C_3 g + C_4 g^2\right], \tag{5}$$

Where g is the factor of form expressed by formula (6):

$$g = \frac{d_{out} - d_{in}}{d_{out} + d_{in}}, \tag{6}$$

C_1, C_2, C_3, C_4 are coefficients given according to the geometrical form shown in Table 5.

Although the accuracy of this expression worsens as the ratio becomes large, it exhibits a maximum error of 8% for $s \leq 3w$. Note that typical practical integrated spiral inductors are built with $s \leq w$. The reason is that a smaller spacing improves the interwinding magnetic coupling and reduces the area consumed by the spiral. A large spacing is only desired to reduce the interwinding capacitance [4].

Table 5
Values of the coefficients used in mohan's formula.

Geometry	C_1	C_2	C_3	C_4
Square	1.27	2.07	0.18	0.13
Hexagonal	1.09	2.23	0	0.17
Octagonal	1.07	2.29	0	0.19
Circular	1	2.46	0	0.2

In Mohan's formula, we consider that the value of inductance is proportional to the product $C_1[ln(C_2) + C_3 + C_4]$ shown in the Table 6.

Table 6
Values of product $C_1[ln(C_2) + C_3 + C_4]$ according to the geometry.

Geometry	Product $C_1[ln(C_2) + C_3 + C_4]$
Square	1.299
Hexagonal	1.0594
Octagonal	1.0614
Circular	1.1

The tables show that the square spiral geometry gives the greatest value of inductance.

III. Conclusion

In this paper, was presented three approximate expressions for spiral inductors of square, hexagonal, octagonal, and circular geometries from [4].The first expression, called the modified Wheeler's expression. This expression is simple and gives very good accuracy. The second expression called Mohan's formula and is obtained by data-fitting techniques. The second expression is derived from electromagnetic principles by approximating the sides of the spiral by current sheets with uniform current distribution. This expression is intuitive and similar in form to inductance expressions for more conventional elements such as coaxial transmission lines and parallel wire transmission lines. All three expressions are accurate, with typical errors of 2–3%, and very simple, and are therefore excellent candidates for use in design and synthesis. [4].

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