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Theoretical basis of the magnetoelastic effect

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***Abstract* — In general, magnetic fields are described by abstract theories which can be hard to grasp as a concept and usually some higher-level math apparatus is required to fully grasp the analyzed concepts. This paper addresses the principle of the magnetoelastic effect with a text explanation which makes the understanding of the principle of the effect less difficult.**

***Keywords* — magnetostriction, magnetic fields, strain, Villari effect.**

I. INTRODUCTION

When it comes to magnetic fields in ferromagnetic materials there are many different effects that can be observed. For example the Joule effect describes the elongation of a material, the Wiedemann effect creates a torque in a ferromagnetic rod that is circularly magnetized which forces the material to twist around its axis. Some effects are exploited in the real world more than others, some are just minor effects that must be compensated for if needed.

The Villari effect (elastomagnetic effect) is a magnetic effect which found lots of use in the real world. From measuring torque exerted on car wheels by an engine, or measuring weights ranging from grams to thousands of tons, the Villari effect enables a means of measurement that is quick and due to the mechanical durability of materials used (iron, nickel, etc.) reliable.

To understand the effect fully a background in higher mathematics is needed, so this paper addresses the description of the origin of the effect mostly in terms of a spoken description that can serve as a learning aid for a wider population.

II. DESCRIPTION OF THE MAGNETIC STATE OF A MATERIAL

When we are analyzing a material from the point of its magnetic properties the most important physical quantity is the magnetic polarization (**I**). It describes the magnetic moment of the material in a unit volume. In general, the magnetic polarization of a material is dependent on the magnetic field strength that the material is placed in. This relation is expressed with equation (1), where μ_0 is the permeability of vacuum, which is a universal constant, and χ is the volume magnetic susceptibility which is a material property. Usually χ is a scalar value (isotropic materials), but in anisotropic materials it must be expressed as a tensor [3].

$$I = \mu_0 \chi H \tag{1}$$

When searching through literature one may find that in many sources the magnetic properties are analyzed via another physical quantity called magnetic flux density (**B**). The relationship between **I**, **B** and **H** is expressed in (2), (3) and (4) where a new quantity is introduced - μ which is the permeability of the material and is also a material property [3].

$$B = \mu_0 H + I \quad (2)$$

$$B = \mu_0 (1 + \chi) H \quad (3)$$

$$B = \mu H \quad (4)$$

A. Magnetization curve

An analytic relationship between **I** and **H** (or **B** and **H**) for magnetic materials is determined by the function of volume susceptibility (χ) on the magnetic field strength (**H**). This function is in most magnetic materials (especially ferromagnetic materials) too complicated to be expressed by a formula and so it is usually expressed graphically. Such graphs are then called magnetization curves. An example of such a curve is shown on Fig. 1 [3].

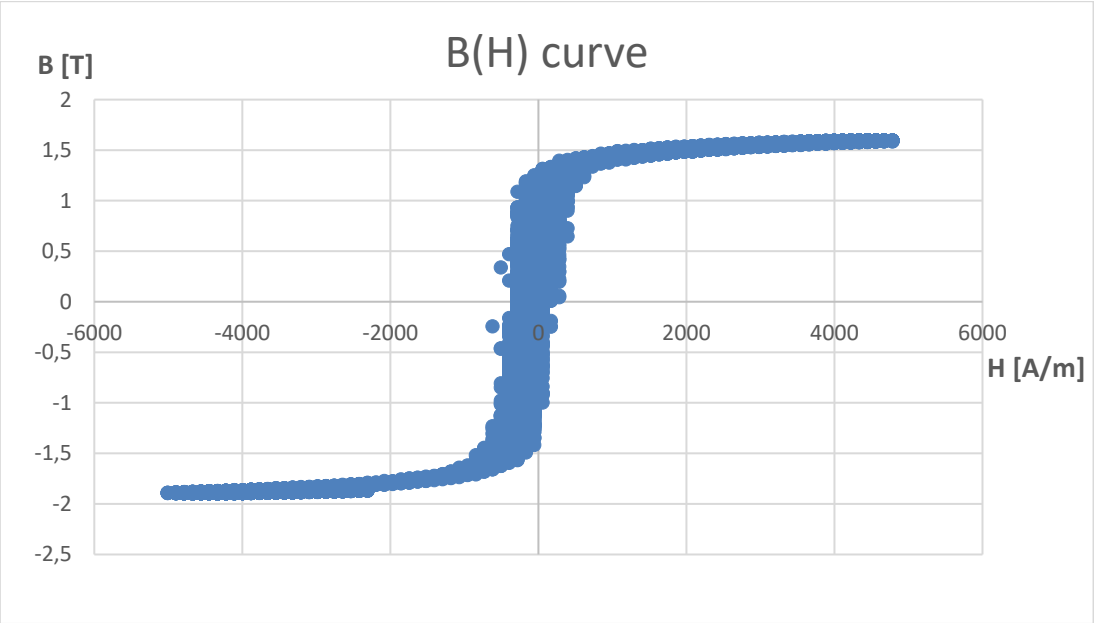

Fig. 1: Magnetization curve (**B** as a function of **H**) measured experimentally. The plot describes the magnetization of a ferromagnetic material consisting of multiple thin (0,35 mm) ferromagnetic sheets magnetically isolated between each other.

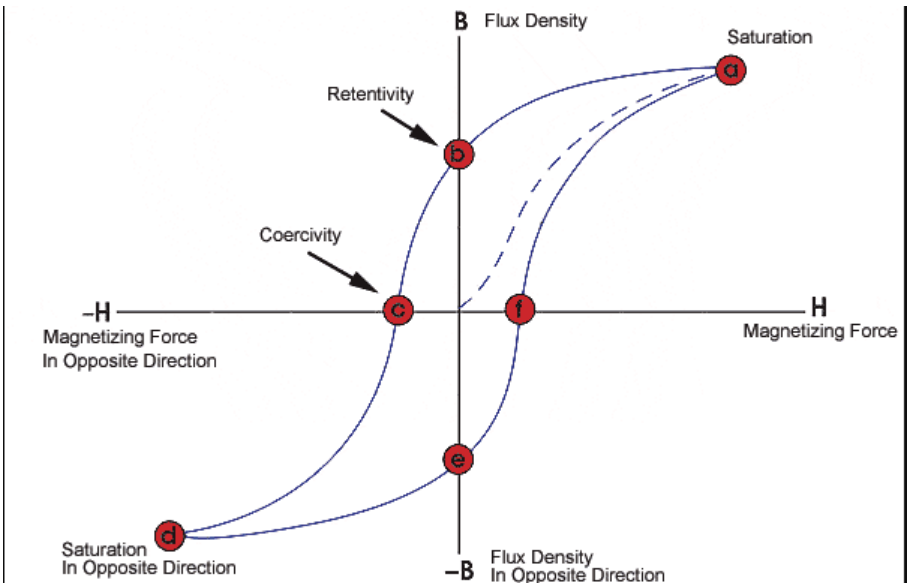

Fig. 2: An example of a magnetization curve (also called a hysteresis loop) where some points are marked which are used to quantify some important magnetic material properties [4].

From the graph of the hysteresis loop some important material constants can be calculated like for example the saturation magnetic flux density (B_S – which is denoted as points *a* and *d* on Fig. 2), then the retentivity or sometimes called remanent magnetic flux density (B_R – which is denoted as points *b* and *e* on Fig. 2) and the coercive force (H_C which is denoted as points *c* and *f* on Fig. 2) [3].

B. Domain structures in ferromagnetic materials

Magnetic materials in general based on the value and sign of their volume magnetic susceptibility (χ) were for a long time divided into 3 groups [3]:

- Diamagnetic materials (χ with typical values around 10^{-5})
- Paramagnetic materials (χ with typical values from 10^{-5} to 10^{-3})
- Ferromagnetic materials (values of χ which are also dependent on **H** and χ with typical values around 10^{6})

This division does not consider important groups of magnetic materials like ferrimagnetic materials (ferrites) and antiferromagnetic materials which are important materials used in engineering (especially ferrites), but since this paper is dedicated to ferromagnetic materials only, this basic division shows the main differences between ferromagnetic and non-ferromagnetic materials (the dependence of χ on **H** and the very large values of χ and therefore also μ) [3].

The existence of a domain structure (which is an ordered structure) can only possible if the forces between atoms with magnetic moments are strong enough to overcome the chaotic molecular movement inside materials. To get a better understanding of the origin of all the organizing forces, let's assume that there exists an effective magnetic field ($\mathbf{H_{eff}}$) which has a strong enough effect that domains can exist [3].

The smallest quantum of the magnetic moment of an electron is called the Bohr magneton denoted as (M_B) which is expressed with the equation below (5), where *e* is the electric charge of an electron, *h* is the Planck constant and m_0 is the mass of an electron [3].

$$M_B = \frac{\mu_0 e h}{4\pi m_0} \tag{5}$$

The energy of the chaotic molecular movement is expressed with equation (6), where *k* is the Boltzmann constant and *T* is the thermodynamic temperature [3].

$$E = kT \tag{6}$$

Ferromagnetic materials retain their properties until a certain temperature called the Curie temperature usually denoted as T_C. It differs between ferromagnetic materials (for example T_C for iron is 1043 K and for Nickel T_C = 636 K), but at this temperature the magnetic structure of these materials vanishes. If we compare the energy of the Bohr magneton which is put in a magnetic field $\mathbf{H_{eff}}$ with the molecular kinetic energy at the Curie point (let's assume a Curie point of 1 000 K), then we get equation (7), from which the needed magnetic field for domain formation can be obtained (8) [3].

$$M_B H_{eff} = kT_C \tag{7}$$

$$H_{eff} = \frac{kT_C}{M_B} = \frac{1{,}38\cdot10^{-23}\cdot10^{3}}{1{,}165\cdot10^{-29}} \doteq 10^{9}\ A/m \tag{8}$$

The value of H_{eff} from (8) is very large. The energy of the Bohr magneton placed in such a strong field is approximately 10^{-20} J. When we consider that two neighboring atoms in a crystal lattice are approximately 10^{-10} m apart and we calculate the resulting energy between two parallel oriented Bohr magnetons, then the resulting energy between them is expressed by (9), where *a* is the distance between the two Bohr magnetons [3].

$$E = \frac{M_B^2}{4\pi\mu_0 a^3} = \frac{\left(1{,}165\cdot10^{-29}\right)^2}{(4\pi)^2\cdot(10^{-10})^3\cdot10^{-7}} \doteq\ 10^{-23}\ J \tag{9}$$

The energy calculated from (9) is approximately 1000 times weaker than is needed for producing an organized structure (the energy of H_{eff} from (8)). This proves (even from experiments made by Dorfman in 1927) that the magnetic domain structure that is seen in ferromagnetic materials has not an origin in the magnetic interactions between atoms in the crystal lattice [3].

The first theory about the forces between elementary particles with a magnetic moment in ferromagnetic materials that in the end led to a spontaneous magnetization of a ferromagnetic body was

created by Weiss in 1907. Weiss hypothesized that the reciprocal forces between magnetic moments could be expressed by a molecular field which is known as the Weiss field. From his theory, he made the following conclusions [3]:

- There exists a spontaneous magnetic polarization when the temperature is lower than a critical value (Curie temperature T_C).
- The spontaneous magnetic polarization is temperature dependent, and the theory provides a very good approximation of the real-world data.
- It explains the paramagnetic behavior of ferromagnetic materials over the critical temperature T_C. [3]

However the theory doesn't provide any information about the qualities of the molecular field which it is based on. It is obtainable only via the theory of quantum mechanics [3].

C. Exchange interaction

In classical physics there is just one interaction between magnetic moments of a system of electrons where the total energy is dependent on the relative orientations of their spins and that is the dipole-dipole interaction, which is however, like it has been shown, too weak to allow the existence of magnetic structures. In quantum mechanics, however, there is (because of a specific property of electrostatically reciprocally coupled electrons) a dependence of the total energy of a system on the total spin moment. Frenkel and Heisenberg both showed independently of each other in 1928, that when a strong electrostatic interaction between electrons is present, an ordered structure can be energetically more advantageous (a lesser energy state) than a disordered one [3].

A further analysis of this interaction revealed that in addition to the energy of the classical Coulomb interaction between two electrons, due to the Pauli principle, there exists also a quantum-mechanical energy contribution, which strongly depends on the reciprocal orientation of the spins of the electrons. The contribution can be expressed with (10), where A is a constant that depends on the distance between the electrons and $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ are the vectors of the spins of the electrons [3].

$$E = -A(S_1, S_2) \tag{10}$$

The interaction that allows for the existence of this energy is called the exchange interaction. The constant A in (10) is called the exchange constant or in some sources because of the means of its calculation, the exchange integral. The theoretical solution of the exchange interaction is very complex, but the approximations of theoretical results show good similarity between the theory and experimental results [3].

III. MAGNETOSTRICTION AND ITS INVERSE EFFECT

A. Magnetostriction

Joule in 1842 did a series of experiments where he observed that a ferromagnetic substance when magnetized changes its length. The changes are very subtle (approx. 1 μm for every meter of material), but using a mechanical contraption consisting of a series of levers he showed that the ferromagnetic material expands in the direction of magnetization and also that the elongation is fast when the material is below saturation (the linear part of the magnetization curve in Fig. 2, between the points *c* and *b*) and as saturation is achieved (the "flat" part of the magnetization curve in Fig. 2, between the points *b* and *a*) then the expansion starts to slow down. An example graph of the elongation of the material in the presence of a magnetic field is shown in the figure below (Fig. 3). We can see from the graph that just as the magnetization curve on Fig. 2 shows some hysteresis (the magnetization depends on the previous state of magnetization), the same applies to magnetostriction [5].

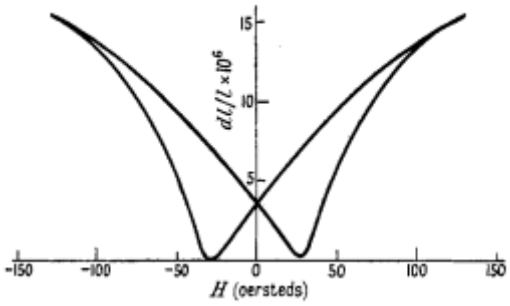

Fig. 3: The magnetostriction of cold-worked nickel during cyclic magnetization as a function of **H** [5]

B. The principle of the magnetoelastic effect

The principle of the magnetoelastic effect (also called the Villari effect or inverse magnetostriction) that is used in the real world is based on the fact that in isotropic ferromagnetic materials (the isotropy here is used as a simplifying factor for the explanation) a change in magnetic properties can be observed when an external stress is applied to them [1].

The applied external stresses deform it, which in turn changes the distances between atoms in the crystal lattice of the ferromagnetic material. Since the distance between atoms is a parameter of the exchange forces in a magnetic domain, the macroscopic effect of this applied stress is a change in magnetic polarization (**I**) or the magnetic flux density (**B**) in the direction of the applied stress (**σ**) when a constant magnetic field strength (**H**) is applied to the material before and during the external stress application. The ferromagnetic material becomes anisotropic, and its permeability can be expressed as a function of the applied stress. This relation is expressed below (11), where μ is the permeability of the material, λ_{ms} is the saturation magnetostriction coefficient, B_s is the value of the magnetic flux density at magnetic saturation of the material and σ is the applied stress [1], [2].

$$\Delta\mu = \frac{2\lambda_{ms}}{B_s^2}\mu^2\sigma \tag{11}$$

This change can be then evaluated as a change of impedance of a coil with a ferromagnetic core or the change of the coupling factor between the primary and secondary of a an iron transformer therefore resulting in a change in the output voltage. The latter has been studied in numerous papers [1].

IV. CONCLUSION

In this paper, we covered the basics of the principle of the Villari effect. Further improvements can be made by a better description of the origins of magnetostriction and elastic constants of the crystal lattice.

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