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The Jiles-Atherton Model of ferromagnetic materials and its dependence on the anhysteretic magnetization

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***Abstract* — This paper addresses the description of the Jiles-Atherton model and one of the most importants parts of the model which is the anhysteretic magnetization. The Jiles-Atherton model is an approach to model the structure and the changes in ferromagnetic materials in the process of their magnetization. The first work was published in 1984 but till this day it is a very popular tool for modelling ferromagnetic hysteresis of isotropic materials. Even though newer methods were created based on the Jiles-Atherton model that enable modelling of anisotropic materials, the original model described in this paper is still one of the most widely used in the field of engineering.**

***Keywords* — Domain wall, Ferromagnetic materials, Hysteresis, Jiles-Atherton, Magnetization**

I. Introduction

Modelling of a response in the form of an internal magnetic flux density to an applied magnetic field intensity in ferromagnetic materials long resisted mathematical rigor. The processes that take part in the magnetization of even an isotropic ferromagnetic material are so complicated that there was a long history of fitting the macroscopic data (like the BH curves) with algebraic expressions that would be sufficient accuracy-wise but yielded no knowledge of why the curves look like the way they do.

Many models that attempted to explain the behavior of the material were created and some of them described the material accurately, but the test of time showed that one of the best models even after 40 years is the Jiles-Atherton model. The model shows an excellent fit with real-world materials, but because for the successful implementation of the model, multiple parameters must be found, like for example the exact form of the anhysteretic magnetization, it is useful to get a theoretical grasp of their meaning, which is the main purpose of this paper.

II. Theory of ferromagnetic hysteresis

The Jiles-Atherton model of ferromagnetic magnetization was created to fill the need of a theoretical approach that could describe the emergence of hysteresis loops when working with ferromagnetic materials. Knowledge back then was sufficient to correctly explain that the hysteresis loop (which is a function of the magnetization of the material on the applied magnetic field intensity) exists because the magnetization process in ferromagnetic materials consists of the movement of domain walls and their bending. In an ideal magnetic material that would be achieved completely reversibly and therefore yield an anhysteretic curve (Fig. *1*), but real-world materials exhibit many kinds of defects (like errors in the crystal lattice of metals) that impede the movement of domain walls and their rotation. Because of these errors the magnetization and the applied field are not in phase and therefore the magnetization lags after the applied field, which yields a hysteresis loop when plotted on a XY plot (Fig. *1*)[2].

A single parameter (denoted *k* in the Jiles-Atherton model) describes this impedance to the motion of domain walls. The model in general therefore is capable to accurately describe the main features of hysteretic materials such as the initial magnetization curve, the saturation magnetization, the coercivity force, the remanent magnetic flux density, and the hysteresis losses[2].

Since the Jiles-Atherton model has been introduced a theoretical foundation was established that enabled to estimate the effects of other factors like temperature and stress on the shape of the hysteresis loop. However it must be stated that till this day there is no general form of the hysteresis loop, but most hysteresis loops that appear in practical applications have similar shapes like the one depicted in the figure below (Fig. *1*), which is a sigmoid. This shape was chosen as the basis of the Jiles-Atherton model and therefore the model describes materials which have this shape of their magnetization curves best[2].

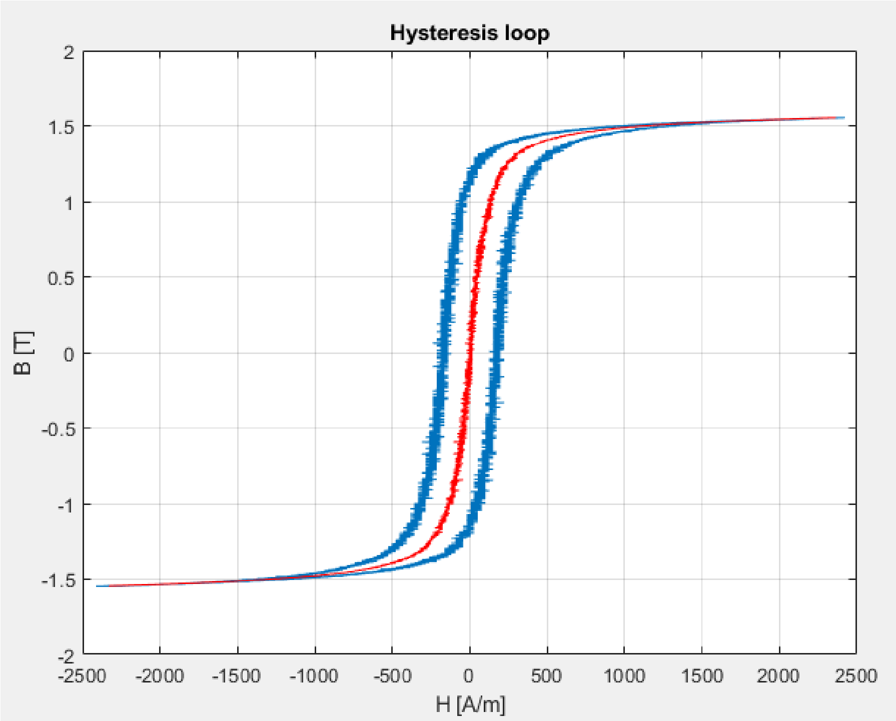

Fig. 1: An experimentally measured B-H hysteresis loop (the blue graph) and the corresponding estimated anhysteretic magnetization curve (the red graph). It can be observed that the anhysteretic curve has no surface area and therefore a material described by such a curve has no hysteresis losses when cyclically magnetized.

Mathematical descriptions of magnetization curves have always evaded generalization, especially the description of ferromagnetic magnetization. Algebraic expressions have been obtained only for high-field magnetization curves for single crystals, high field magnetization of polycrystals and low field magnetization curves for polycrystalline materials which exhibit Rayleigh loops. An algebraic expression for a whole magnetization curve which also incorporates saturation could be produced to an arbitrary degree of accuracy, but that would evade insight into the theoretical basis as the expression would only serve as a best fit for measured data[2].

Many competing theories to the Jiles-Atherton model have been established and some give good results for specific materials, but they usually exhibit some problems and limitations. For example Globus and Duplex[3] developed a model where the magnetization is also based on domain wall movements inside the material which is like the Jiles-Atherton model in concept, but the assumption here is that the domain wall can experience translational impedance only on the outer boundary of ferromagnetic grains. In the real world however, a domain wall can be pinned also to any inhomogeneities inside the grain as the mentioned vacations in the crystal lattice or areas of inhomogeneous stress variations. A similar problem is observed in the form of Porteseil and Vergne[4] which also studied the behavior of magnetization due to the domain wall movements, but they took into account only the irreversible movement of domain walls, whereas a real material exhibits also a reversible domain wall movement in the form of domain wall bulging[2].

When developed, the Jiles-Atherton model produced the typical sigmoid-shaped hysteresis loops which acquire their shapes mostly because of the pinning sites which impede domain wall movement. In the model there is no distinction between different types of pinning sites, but rather there is an average energy of a single pinning site and a uniform distribution of them through-out the material is assumed. Such assumptions suggest that the material is isotropic, but more complex models using the Jiles-

Atherton models have been created, which enable the description of anisotropic materials. The basic overview of one of the most important model properties is described in the following pages [2].

A. Anhysteretic magnetization curve

The Jiles-Atherton model requires a function that would describe the anhysteretic magnetization of a ferromagnetic material. Because the model is built around the idea of the state of individual domains during the magnetization process let's consider the energy per unit volume of a domain which has a total magnetic moment *m* and experiences a magnetic field intensity *H*. If the magnetic domain is isotropic and therefore has no preferred orientation of magnetization, then (1) describes the energy of the domain per unit volume, where μ_0 is the permeability of vacuum, *m* is the magnetic moment vector and *H* is the magnetic field intensity vector (the field that the domain experiences, not the external applied one)[2].

$$E = -\mu_0 \cdot \boldsymbol{m} \cdot \boldsymbol{H} \tag{1}$$

Because magnetic domains create the internal structure of a ferromagnetic material, there will be some coupling between the domains. The magnetization vector of the domain adds to the external magnetic field intensity so when we express the coupling of the domain to the magnetization of the material, formula (2) is defined, where *M* is a vector of magnetization of the material and α represents the interdomain coupling, which is determined experimentally[2].

$$E = -\mu_0 \cdot \boldsymbol{m} \cdot (\boldsymbol{H} + \alpha \boldsymbol{M}) \tag{2}$$

So when we define the effective field (H_e) which the domain experiences as a superposition of the external magnetic field intensity and the magnetization of the material (3), and equation (2) is rewritten in terms of this effective field we get equation (4)[2].

$$\boldsymbol{H_e} = \boldsymbol{H} + \alpha \boldsymbol{M} \tag{3}$$

$$E = -\mu_0 \cdot \boldsymbol{m} \cdot \boldsymbol{H_e} \tag{4}$$

We can think of the magnetization as a function of this effective field H_e, therefore a function $f(H_e)$ can be defined for equation (5), where M_S is the saturation magnetization of the material and is a material constant. The function can be arbitrary and the only constraints the function *f* must follow is that when its input is 0, then the output is also 0 and when the input tends to infinity then the output converges to 1. This expression describes a ferromagnetic material in its equilibrium state which exists only in a perfect ferromagnetic material with no domain wall movement impedance. Such a curve can be measured by introducing a decaying AC field to a DC magnetic field with a magnetic field intensity *H*. When the AC field is completely decayed, then the magnetization of the material remains at a value of M_0 which is point of the anhysteretic curve. The measurement process is more closely described in [5]. The anhysteretic magnetization curve is then expressed by (6)[2].

$$M = M_s \cdot f(H_e) \tag{5}$$

$$M_{an}(H_e) = M_s \cdot f(H_e) \tag{6}$$

Usually the function *f* is the modified Langevin function $L(H_e)$ which therefore leads the anhysteretic curve to be expressed by formula (7), where the parameter *a* defines the shape of the magnetization curve[2].

$$M_{an}(H_e) = M_s \left(\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right) \tag{7}$$

This anhysteretic magnetization formula is what is the backbone of the whole theoretical approach of the model. Any inaccuracy in the anhysteretic model will result in inaccuracies in the whole model. Every other part (the reversible and irreversible changes in magnetization) just builds upon it. The whole hysteresis loop mathematical model was described in the original paper[2].

B. Types of anhysteretic magnetization curves

The theory of ferromagnetism was largely based on the study of paramagnetic materials. The assumption was made that in comparison to paramagnetic materials the exchange forces between adjacent atoms in ferromagnetic materials are strong enough to form highly organized structures that are now called ferromagnetic domains. One of the most used functions which describe the dependence of the magnetic field on the state of the material is the Brioullin function (8), which expresses the magnetic field dependence on the total angular momentum *J* of the material, which is a quantum number that can only attain discrete values (0; ½; 1; 3/2; 2;…)[8].

$$\mathfrak{B}(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{1}{2J} x\right) \tag{8}$$

When we take the classical limit in which J tends to infinity, then the Brioullin function can be defined as the Langevin function which was described above, which is defined by formula (9). The derivation is shown in [8].

$$\lim_{J\to\infty} \mathfrak{B}(x) = \mathcal{L}(x) = \coth\left(x - \frac{1}{x}\right) \tag{9}$$

Here we can see that choosing the Langevin function for the anhysteretic magnetization formula could seem random, but it has a theoretical basis. However, different materials with different properties might fit better with hysteresis models based on different anhysteretic magnetization curves. The other most used ones are the erf-based model, the exp (exponential function) based model and the arctan function-based model, which is expressed by formula (10). Every type of curve fits best to ferromagnetic materials with a different internal structure (polycrystalline, amorphous, nanomaterials, etc.). Many studies have been conducted to find the best fit for specific material types. [9]

$$M(H) = M_s \cdot \frac{2}{\pi} \cdot \arctan\left(\frac{H_e}{a}\right) \tag{10}$$

Some studies also introduce a model where the anhysteretic magnetization is described by a superposition of the usual "single" magnetization formulas. One example is the double-Langevin function which consists of the sum of two Langevin functions (11), which in this case doesn't formulate the relationship between the effective field H_e and the magnetization M, but with the magnetic polarization J. The relationship between M and J is expressed in (12). It consists of two parts. The two parts denote the reversible and irreversible changes of magnetization well and so their sum can be used to approximate some materials' magnetic properties. A study that showed materials which are described by the double-Langevin function very well is in [7].

$$J_{an}(H) = J_{s,a}\left[\coth\left(\frac{H}{H_{0,a}}\right) - \frac{H_{0,a}}{H}\right] + J_{s,b}\left[\coth\left(\frac{H}{H_{0,b}}\right) - \frac{H_{0,b}}{H}\right] \tag{11}$$

$$J = M \cdot \mu_0 \tag{12}$$

Usually a whole system is just as efficient as the weakest part of it. That is why a great deal of pedantry must be shown when fitting and choosing a function to describe the anhysteretic magnetization curve. The functions mainly differ in the shape of the curve in the so-called "knee" of the hysteresis loop just before magnetic saturation which can affect the accuracy of the model when the materials used for the cores of inductors or transformers are driven into saturation during their operation.

III. Conclusion

The Jiles-Atherton model has been briefly described especially from the view of the ahysteretic magnetization curve. The model requires multiple parameters which must be carefully selected, because on some of them, physical constraints are placed for the model to be able to provide a realistic hysteresis model. Nowadays many algorithms can be found that attempt to find the parameters by numerical algorithms that fit the experimental curves with the modelled ones. They vary in accuracy and computational cost. One of the biggest disadvantages of the model is that it is very sensitive on the shape of the anhysteretic curve, which by itself is a very difficult entity to measure experimentally with sufficient accuracy. The Jiles-Atherton model usually implements this curve via a modified Langevin function, but any function can serve as the anhysteretic curve if it fits experimental data, but there are some most commonly used functions which were described in this paper.

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